

# STATISTICAL ANALYSIS OF MODELS OF LIQUID MIXING IN VARIOUS HYDRODYNAMIC CONDITIONS

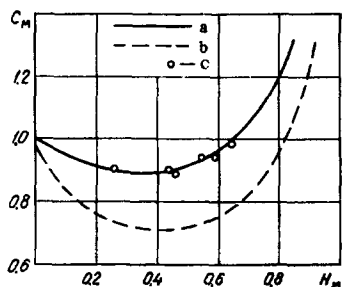
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Inzhenerno-Fizicheskii Zhurnal, Vol. 11, No. 3, pp. 378-381, 1966

UDC 66.063.8

A statistical analysis has been made of the distribution functions for residence time of a liquid in different flow formulations. The region of application of the different models of longitudinal dispersion is discussed.

In the flow of a liquid longitudinal dispersion (mixing) of the particles in the direction of their motion occurs. If we suddenly inject a batch of labeled particles at time  $t = 0$  at the inlet of a channel of length  $l$ , they will emerge from the channel at different times. Their concentration at the outlet is determined by the hydrodynamics of the flow.



Dependence of the probability density of the distribution mode  $C_M$  on the value of the mode  $H_M$  for a finite channel: a) according to the diffusion model; b) according to the cell model; c) the experimental points.

Numerous mathematical models [1-7] have been put forward to describe the mixing.

A thorough verification and elucidation of a model should help to reveal the true mechanism of longitudinal dispersion, and to describe the concentration distribution on that basis.

Such a verification has been accomplished in this work on the basis of the statistical characteristics of the distribution curve of time of residence of the liquid particles.

We have determined the probability density of a mode  $C_M$ , the initial moments  $\nu_m = \int_0^{\infty} CH^m dH$ , and the dispersion  $\mu_2 = \int_0^{\infty} C(H - \nu)^2 dH$ .

If the model accurately describes the mixing process, then the values of its parameter ( $B$  for the diffusion model, or  $n$  for the cell model), as found from the values of the various statistical characteristics, must coincide, or give a group of numbers close to one another.

The theoretical dependences of the statistical characteristics of distribution functions on the values of the Bodenshtein parameter  $B$  and on the number of cells  $n$  for complete mixing have been given in [8].

The most typical kinds of flow for mass transfer and reaction equipment are: flow of a single-phase stream, flow over a packed bed, flow on a plate, and flow in a tall bubble bed. For the first three cases the distribution curves have been published, while for the last case the curves are not in the literature, and we conducted experiments.

**Tall Bubble Column.** Tests were done in a column of diameter 54.5 mm and total height 230 cm. Water from a constant level header tank came through a rotameter to an annular spreader, and from there to the column. Gas came through a flowmeter, entered below the distributing grid, and passed through the column, which had 22 apertures of diameter 1 mm. The water left the column through three side pipes in each of which was mounted platinum mesh to measure the electrical conductivity of the solution.

To accomplish an impulsive action a measured amount of NaCl solution was poured into a container, which was closed underneath by a disk lapped to fit the bottom face of the container, and pressed against it by springs. The top of the container was connected through a three-way valve either to vacuum or to a nitrogen line. At the time when the impulse was required, the container was connected to the nitrogen line, and a batch of the electrolyte was very rapidly injected into the column. The outlet function was recorded by an electronic bridge as a curve of electrical conductivity of the emerging solution with time. This curve was then converted to a curve of concentration with time,  $c(t)$ , and, further, to a curve of probability density of residence time of the liquid particles in the column  $C$  on the time  $H$ , by the formulas given above.

The flow rate of the liquid was 7.5-15.5 ml/sec, and of the gas 6.5-46.0 ml/sec, while the height of the bed was varied over the range 86.5-216.5 cm.

The table gives the results of one of the tests. The flow rate of liquid was 11.6 ml/sec, of gas 6.8 ml/sec, and the height of the bed 216.5 cm.

It follows from the table that the deviation of the cell model parameter,  $n$ , as found from the various statistical characteristics,  $\nu_2, \nu_3, \mu_2$  and  $C_M$ , is several times greater than that of the values of the diffusion model parameter,  $B$ , as found from the same statistical characteristics.

The graph showing the theoretical dependences of the quantity  $C_M$  on the mode  $H_M$  for both models, as

well as the experimental points, also speaks convincingly in favor of the diffusion model.

**Flow in a Cylindrical Tube.** We have taken the experiment of Taylor and Brown [9] from reference [10]. The location of the impulsive injection and the location of the measurement of concentration are at large distances from the ends of the tube, and we therefore compare the results with the theoretical solution for an infinite channel.

From analysis of the results presented in the table, we come to the conclusion that in this case one obtains a close group of values of B, thus ensuring that the theoretical values of  $\nu_2, \nu_3, \nu_4, \mu_2$  and  $C_M$  are equal to their experimental values.

**Flow of a Liquid over a Packing Element.** The packing element taken in [11] was a bed of 128 spheres of diameter 3.8 cm. We conducted 15 tests with this arrangement, and the majority proved to be close to the diffusion model of mixing. For a fluid flow rate of 1.98 ml/sec the results are given in the table; the relative deviation of the cell model parameter is twice as large as that of the quantity B. In the rest of the tests we obtained values in a close group for both B and n, although the deviation of the cell number n was larger, as a rule. Such a considerable difference between the diffusion and cell models in this region is due to the fact that for large values of the parameter (B; n), both models lead to distribution functions that are close together, while their statistical characteristics tend to a common limit for both models:  $\nu_2, 3, 4, \rightarrow 1; \mu_2 \rightarrow 0, C_M \rightarrow \infty$ .

**Flow of a Liquid on a Perforated Plate.** A residence time distribution function has been developed in a similar way for liquid on a perforated plate, and has been published in [12].

It may be seen from the table that neither the cell model nor the diffusion model can represent with sufficient completeness the basic properties of the distribution functions: the discrepancy of values of B and n, as found from the various statistical parameters, exceeds 42%.

Analysis of the experiments of [13] leads to similar results.

However, values of the zero moment  $\nu_0$  differ comparatively greatly from unity (0.96), which indicates a low experimental accuracy.

It has thus been established that the diffusion model of mixing is suitable to describe the process of longitudinal dispersion in flow of a liquid in a tall bubble column, in the flow of a single-phase stream in a cylindrical tube, and in flow over a bed of spheres.

The question of the nature of mixing of a liquid on a perforated plate requires further investigation, since the available experimental data are not accurate enough.

NOTATION

C is the probability density,  $C = c/c_0$ ;  $c_0$  is mean concentration; H is relative time,  $H = t/\tau$ ;  $\tau$  is the

mean time; m is the order of the initial moment,  $m = 1, 2, 3, 4$ ; B is the Bodenshtein parameter,  $B = l/D\tau$ ; D is the longitudinal diffusion coefficient; n is the cell number for complete mixing.

Experimental Values of Statistical Characteristics, and Values of the Parameters B and n Computed Therefrom

Characteristics and parameters	for a tall bubble column	for a cylindrical tube	for flow over a packing element	for cross flow on a perforated plate
$\nu_2$	1.346	2.865	1.0597	1.0995
$\nu_3$	2.388	7.516	1.1963	1.4439
$\nu_4$	5.169	22.604	1.4403	2.1487
$\mu_2$	0.3659	0.8144	0.0593	0.1733
$C_M$	0.950	0.637	1.86	1.31
$B_{\nu_2}$	4.515	4.61	32.4	19.0
$B_{\nu_3}$	4.468	4.66	31.4	14.2
$B_{\nu_4}$	4.652	4.89	30.8	13.4
$B_{\mu_2}$	4.15	4.59	32.7	10.4
$B_{C_M}$	4.30	5.35	37.6	15.5
$n_{\nu_2}$	2.89	2.94	16.9	10.1
$n_{\nu_3}$	2.70	2.78	16.0	7.4
$n_{\nu_4}$	2.64	2.74	15.1	6.8
$n_{\mu_2}$	2.73	2.95	16.9	5.76
$n_{C_M}$	4.65	5.12	22.0	9.90

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19 April 1966

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